## Day 15

Kinematics of Wheeled Robots

## Wheeled Mobile Robots

- robot can have one or more wheels that can provide
> steering (directional control)
- power (exert a force against the ground)
- an ideal wheel is
p perfectly round (perimeter $2 \pi r$ )
- moves in the direction perpendicular to its axis


## Wheel




## Deviations from Ideal

This illustration gives a good sense of the steering and throttling you'll have to input to keep your car drifting. When snapping the car from its full drift angle in one direction to full drift in the opposite direction, be prepared for the rear end to come around with more force then when initiating a drift from straight-ahead running. Give yourself plenty of space as you master your technique so you don't slap a curb or something equally immobile!

1. Steer hard into the turn to initiate a slide, then countersteer before the car loops out.

2] continue countersteering to maintain the slide. It's a balancing act!


## Instantaneous Center of Curvature

- for smooth rolling motion, all wheels in ground contact must
- follow a circular path about a common axis of revolution
- each wheel must be pointing in its correct direction
- revolve with an angular velocity consistent with the motion of the robot
b each wheel must revolve at its correct speed


## Instantaneous Center of Curvature



## Castor Wheels

provide support but not steering nor propulsion

(a) Castor Wheel

(b) Rollerball wheel

## Tangent Bug



## Differential Drive

two independently driven wheels mounted on a common axis


## Differential Drive

velocity constraint defines the wheel ground velocities

$$
\begin{aligned}
& v_{r}=\omega\left(R+\frac{\ell}{2}\right) \\
& v_{\ell}=\omega\left(R-\frac{\ell}{2}\right)
\end{aligned}
$$

given the wheel ground velocities

$$
\begin{aligned}
& R=\frac{\ell}{2} \frac{\left(v_{r}+v_{\ell}\right)}{\left(v_{r}-v_{\ell}\right)} \\
& \omega=\frac{\left(v_{r}-v_{\ell}\right)}{\ell}
\end{aligned}
$$

## Forward Kinematics

for a robot starting with pose $\left[\begin{array}{lll}0 & 0 & 0\end{array}\right]^{\mathrm{T}}$ moving with velocity $V(t)$ in a direction $\theta(t)$ :

$$
\begin{aligned}
& x(t)=\int_{0}^{t} V(t) \cos (\theta(t)) d t \\
& y(t)=\int_{0}^{t} V(t) \sin (\theta(t)) d t \\
& \theta(t)=\int_{0}^{t} \omega(t) d t
\end{aligned}
$$

## Forward Kinematics

for differential drive:

$$
\begin{aligned}
& x(t)=\frac{1}{2} \int_{0}^{t}\left(v_{r}(t)+v_{\ell}(t)\right) \cos (\theta(t)) d t \\
& y(t)=\frac{1}{2} \int_{0}^{t}\left(v_{r}(t)+v_{\ell}(t)\right) \sin (\theta(t)) d t \\
& \theta(t)=\frac{1}{\ell} \int_{0}^{t}\left(v_{r}(t)-v_{\ell}(t)\right) d t
\end{aligned}
$$

## Sensitivity to Wheel Velocity

$$
\begin{aligned}
& v_{r}(t)=1+\mathcal{N}\left(0, \sigma^{2}\right) \\
& v_{\ell}(t)=1+\mathcal{N}\left(0, \sigma^{2}\right) \\
& \theta(0)=0 \\
& t=0 \ldots 10 \\
& \ell=0.2
\end{aligned}
$$



