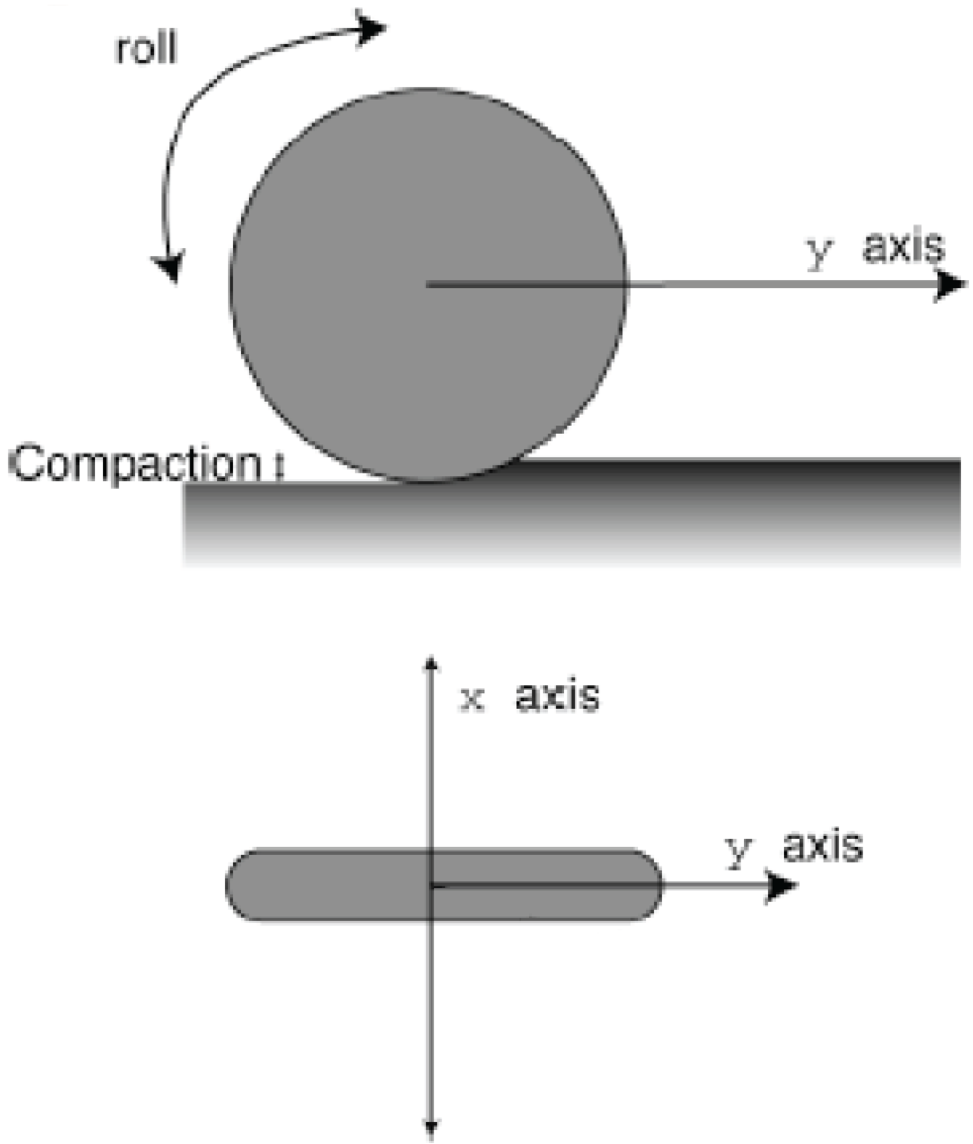
Day 15

Kinematics of Wheeled Robots

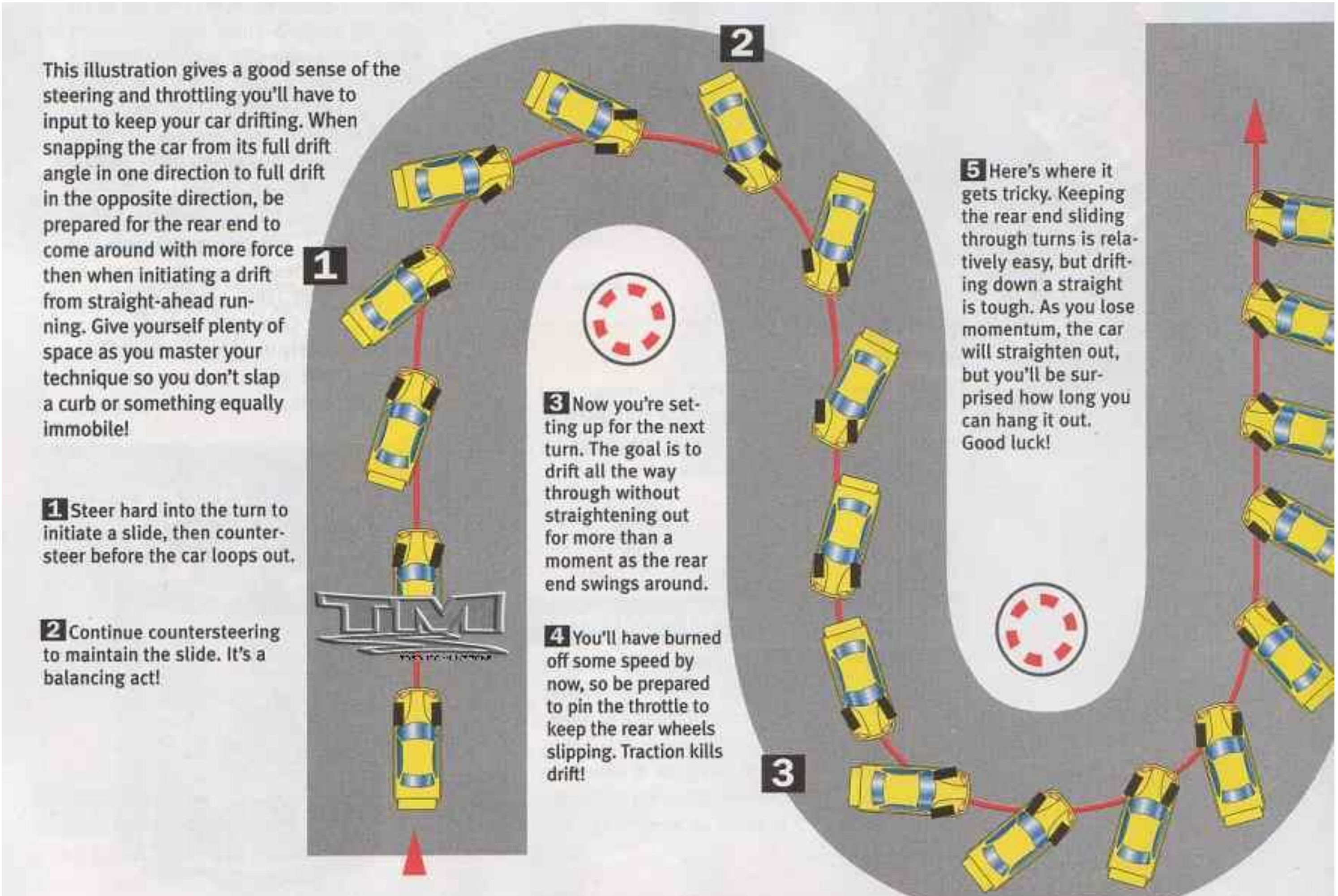
Wheeled Mobile Robots

- robot can have one or more wheels that can provide
  - steering (directional control)
  - power (exert a force against the ground)
- an ideal wheel is
  - perfectly round (perimeter $2\pi r$)
  - moves in the direction perpendicular to its axis

Wheel



Deviations from Ideal

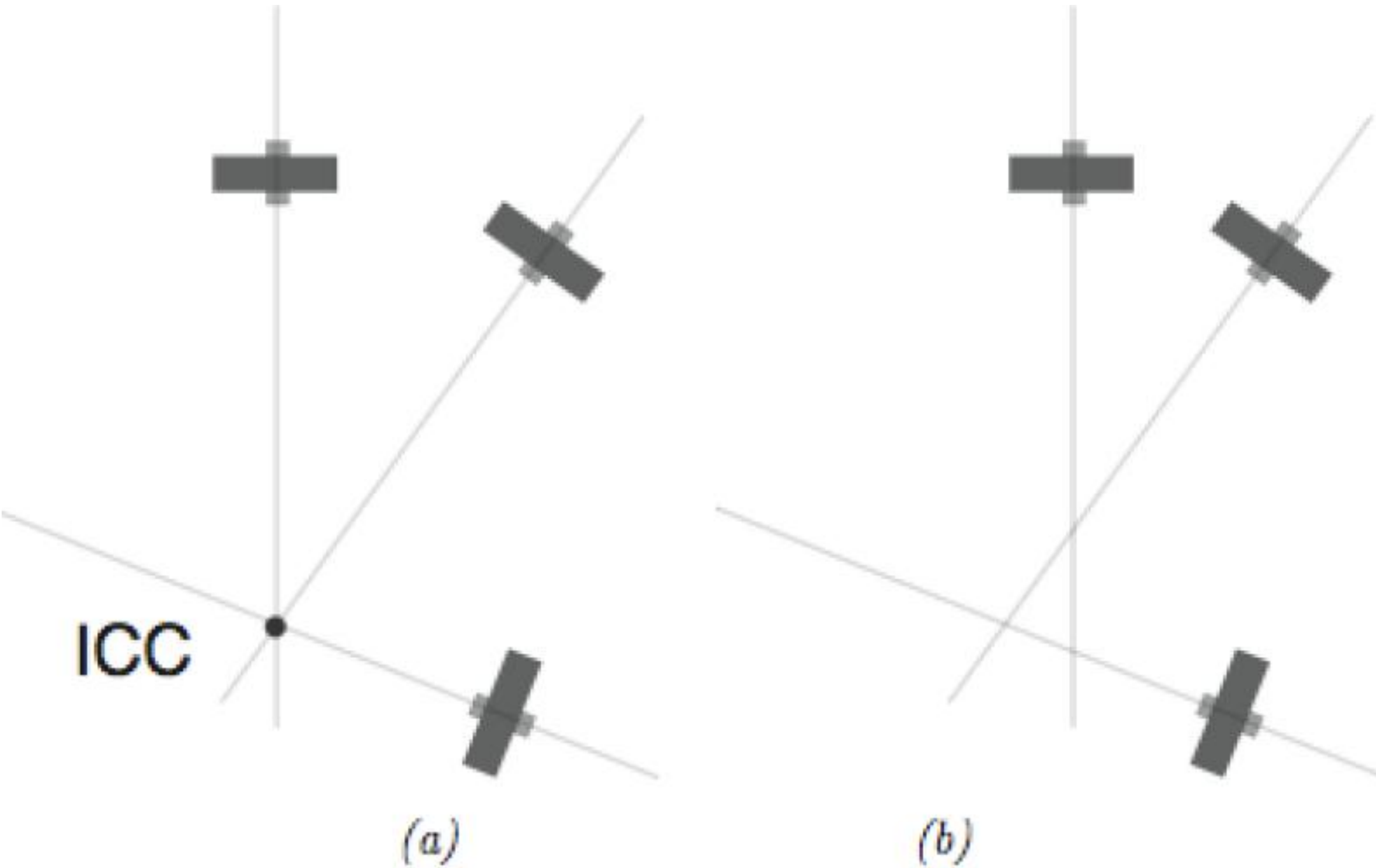


http://worlddrifting.com/wp-content/uploads/2010/02/drifting-techniques.jpg

Instantaneous Center of Curvature

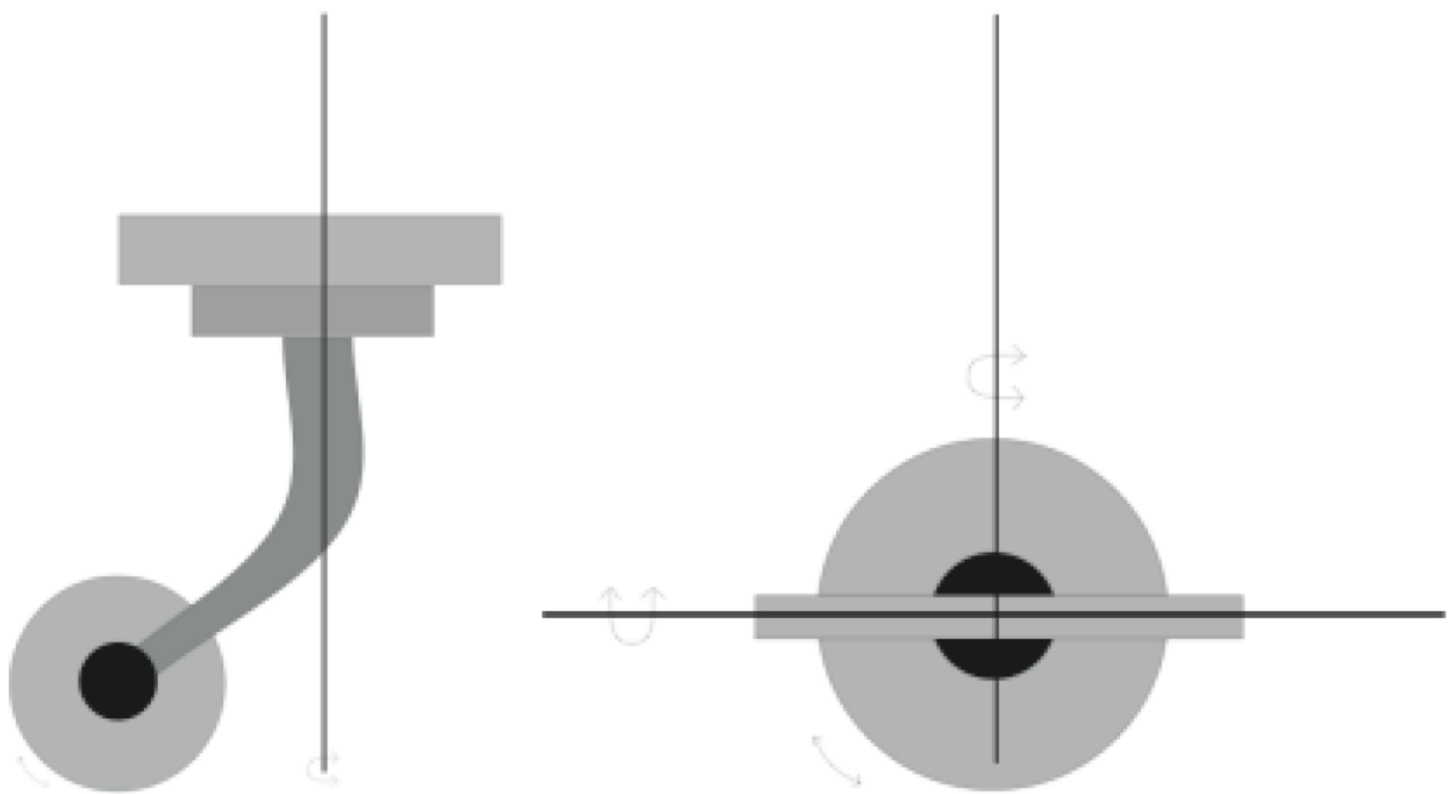
- for smooth rolling motion, all wheels in ground contact must
  - follow a circular path about a common axis of revolution
    - each wheel must be pointing in its correct direction
  - revolve with an angular velocity consistent with the motion of the robot
    - each wheel must revolve at its correct speed

Instantaneous Center of Curvature



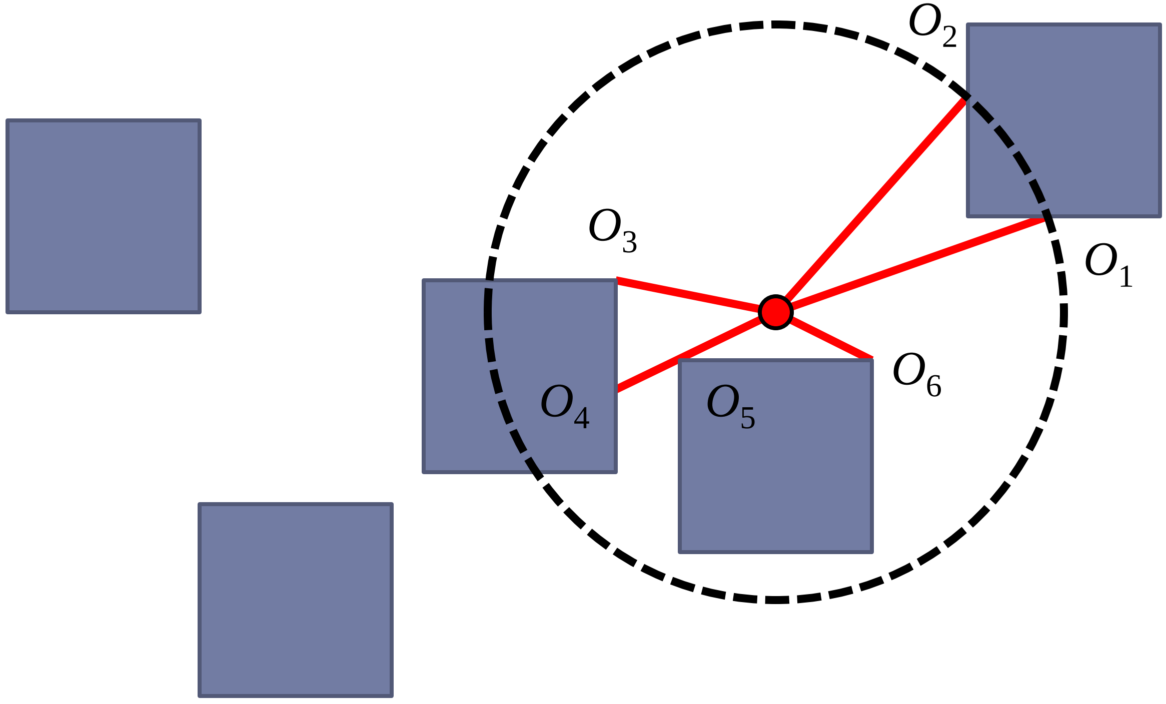
Castor Wheels

- provide support but not steering nor propulsion



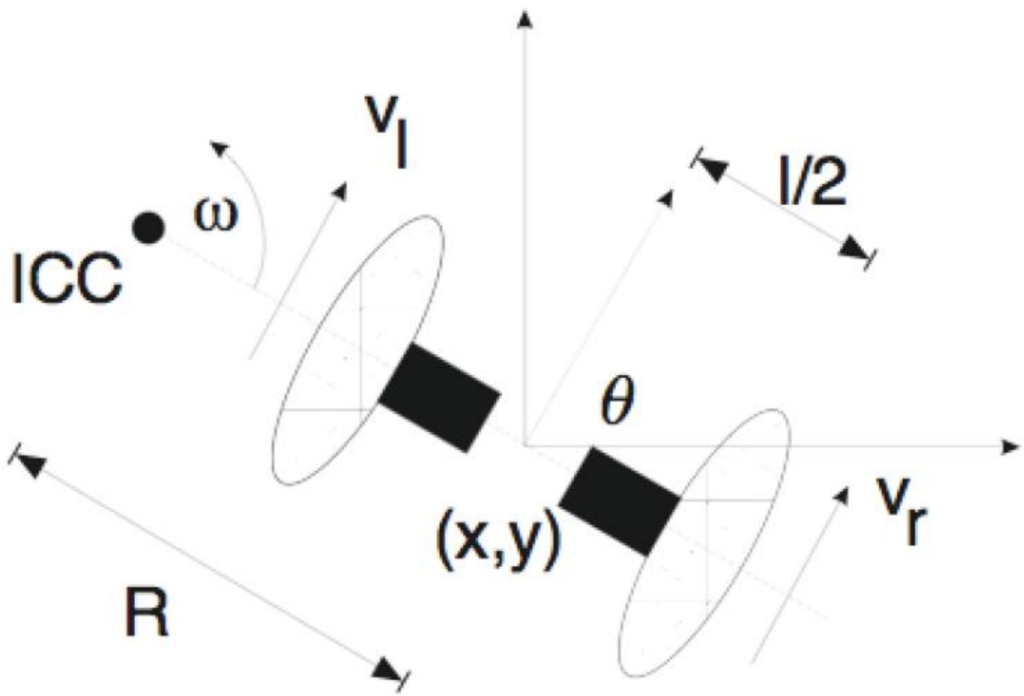
(a) Castor Wheel (b) Rollerball wheel

Tangent Bug

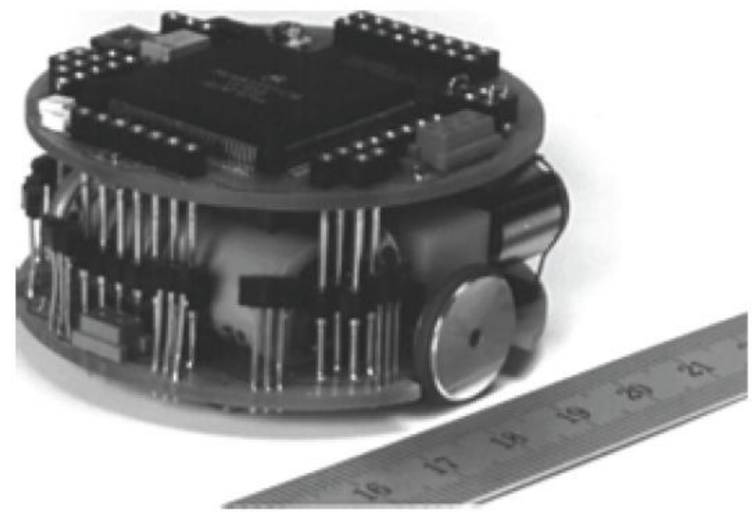


Differential Drive

- two independently driven wheels mounted on a common axis



(a) Differential Drive


(b) Khepera Robot

Differential Drive

- velocity constraint defines the wheel ground velocities

$$v_r = \omega(R + \frac{\ell}{2})$$

$$v_\ell = \omega(R - \frac{\ell}{2})$$

- given the wheel ground velocities

$$R = \frac{\ell}{2}\frac{(v_r + v_\ell)}{(v_r - v_\ell)}$$

$$\omega = \frac{(v_r - v_\ell)}{\ell}$$

Forward Kinematics

- for a robot starting with pose $[0\ \ 0\ \ 0]^T$ moving with velocity $V(t)$ in a direction $\theta(t)$:

$$x(t) = \int_0^t V(t)\cos(\theta(t))\, dt$$

$$y(t) = \int_0^t V(t)\sin(\theta(t))\, dt$$

$$\theta(t) = \int_0^t \omega(t)\, dt$$

Forward Kinematics

- for differential drive:

$$x(t) = \frac{1}{2}\int_0^t (v_r(t) + v_\ell(t))\cos(\theta(t))\, dt$$

$$y(t) = \frac{1}{2}\int_0^t (v_r(t) + v_\ell(t))\sin(\theta(t))\, dt$$

$$\theta(t) = \frac{1}{\ell}\int_0^t (v_r(t) - v_\ell(t))\, dt$$

Sensitivity to Wheel Velocity

$$v_r(t) = 1 + \mathcal{N}(0, \sigma^2)$$

$$v_\ell(t) = 1 + \mathcal{N}(0, \sigma^2)$$

$$\theta(0) = 0$$

$$t = 0 \ldots 10$$

$$\ell = 0.2$$